MAT 150A HW08

[add your name here]

Due Tuesday, 3/5/24 at 11:59 pm on Gradescope

Proof-based course This is a proof-based course and you are expected to **clearly prove** all your claims. If you're wondering how much detail to include, a good rule of thumb is that your proofs should be slightly more detailed than the proofs in the book, but not less detailed. They should also not be unreasonably verbose.

Reminder Homeworks must be typed using LaTeX **in full sentences with proper mathematical formatting**. Handwritten homeworks will not be accepted. If there is a documented reason why you can't type up your homework, let me know and we can discuss an alternate policy. Otherwise, please consider learning how to properly write and typeset mathematics as part of this course.

Exercise 1

(Exam 2 Review) Suppose $H \leq G$ is not a normal subgroup. Prove that there exist left cosets aH and bH such that their product $(aH)(bH)$ is not a coset of H.

Exercise 2

(Exam 2 Review) Let $G = (\mathbb{R}^2, +)$ and let $D \leq G$ denote the set of points on the diagonal:

$$D = \{(x, y) \in \mathbb{R}^2 \mid y = x\}.$$

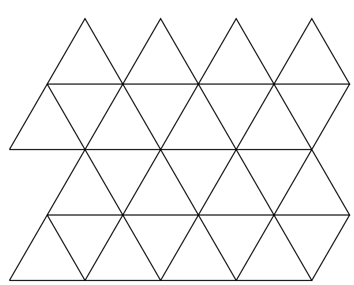
Briefly explain why $D \trianglelefteq G$. Then use the First Isomorphism Theorem to identify the quotient group G/D with a familiar group.

Exercise 3

(Exam 2 Review) Let m be an orientation-reversing isometry. Prove **algebraically** that m^2 is a translation.

Exercise 4

Let G denote the group of symmetries of the following **infinite** wallpaper pattern P constructed from equilateral triangles of side length 1:



(a) Determine the point group $\overline{G}$ of G, and find the index in G of the subgroup of translations L.

(b) Find translation vectors $a, b \in \mathbb{R}^2$ realizing L as the lattice $\mathbb{Z}a + \mathbb{Z}b$.

Exercise 5

Let $G = D_4$ be the dihedral group of symmetries of the square $[-1,1]^2 \subset \mathbb{R}^2$, generated by $\rho = \rho_{\pi/2}$ and τ reflection across the e_1-axis.

(a) What is the stabilizer of the vertex $v = (1,1)$?

(b) What is the stabilizer of the top edge e connecting $(-1,1)$ and $(1,1)$?

(c) G operates on the set of two elements consisting of the two diagonal lines. What is the stabilizer of a diagonal?

You do not need to give a full proof of your answers; give a brief explanation to support your answer. For example, my answer for part (a) begins with "By inspection, the elements of D_4 that fix v are ..." and then identifies the physical meaning of the elements.