MAT 250B HW07

[add your name here]

Due Friday, 2/23/24 at 11:59 pm on Gradescope

Exercise 1

Recall that an abelian group A is **torsion-free** if it has no nonzero elements of finite order. Prove that the following conditions are equivalent for an abelian group A:

(i) A is torsion-free and divisible.

(ii) For every $n \in \mathbb{N}$[1], the multiplication map $\mu_n : A \to A$, $a \mapsto na$, is an isomorphism.

(iii) A is a vector space over $\mathbb{Q}$.

Exercise 2

Let A be a torsion-free abelian group. Prove that A can be embedded as a subgroup of a vector space over $\mathbb{Q}$. *Hint: Embed A in a divisible abelian group D, and then show that $A \cap T(D) = \{0\}$, where $T(D)$ is the torsion subgroup of D.*

Exercise 3

Let $\{A_i\}_{i\in I}$ be an arbitrary collection of right R-modules. Prove that $\bigoplus_{i\in I} A_i$ is flat if and only if every A_i is flat.

Exercise 4

In this exercise you may use the following proposition:

Proposition. Let k be a PID. Then any submodule of a free k-module is also free.

(a) Prove that over a PID, a module M is projective if and only if it is free.

(b) Prove that as a $\mathbb{Z}$-module, $\mathbb{Q}$ is flat but not projective.

[1] where $\mathbb{N} = \mathbb{Z}_{>0}$