

MAT 215A HW01

Submit your **typeset** solutions to Gradescope by 9:00 pm on the Thursday when this problem set is due; see the class calendar for due dates.

Exercises pulled directly from Hatcher are based on the online version of the book linked to from the course website.

I will secretly choose two of these problems to be graded for accuracy. The remainder will be graded for completion.

1. (a) Give an explicit homeomorphism from \mathbb{R}^n to the open unit disk $\text{int}(D^n)$.
(b) Construct an explicit deformation retraction of \mathbb{R}^n onto the origin 0.
(c) Construct an explicit homotopy between the maps $f, g : \mathbb{R} \rightarrow \mathbb{R}^2$ where

$$f(x) = (x, |x|) \quad \text{and} \quad g(x) = (x, x^2).$$

- (d) Construct an explicit deformation retraction of $\mathbb{R}^n - \{0\}$ onto the unit sphere S^{n-1} .
2. HE 0.3 (Hatcher Chapter 0 Exercise 3)
3. HE 0.4
4. HE 0.5
5. HE 0.6
6. HE 0.14
7. (a) Prove that $\mathbb{R}P^n = A \sqcup B$ where $A \cong \mathbb{R}P^{n-1}$ and $B \cong \mathbb{R}^n$.
(b) Use (a) to inductively prove that $\mathbb{R}P^n = A_0 \sqcup A_1 \sqcup \cdots \sqcup A_n$ where $A_k \cong \mathbb{R}^k$.
(c) Using your decomposition from (b), find a cell decomposition of $\mathbb{R}P^n$, specifying the number of k -cells for each k and writing down the gluing maps $f_k : S^{k-1} \rightarrow X^{k-1}$.