

MAT 215A HW07

Submit your **typeset** solutions to Gradescope by 9:00 pm on the Thursday when this problem set is due; see the class calendar for due dates.

Exercises pulled directly from Hatcher are based on the online version of the book linked to from the course website.

I will secretly choose two of these problems to be graded for accuracy. The remainder will be graded for completion.

1. Prove that covering space (10) of $S^1 \vee S^1$ on page 58 is not normal.
2. (HE 1.3.12, *originally on HW05*) Let a, b be the generators of $\pi_1(S^1 \vee S^1, x_0)$ where x_0 is the wedge point.
 - (a) Draw a picture of the covering space $\tilde{X}$ of $S^1 \vee S^1$ corresponding to the normal subgroup K generated by $a^2, b^2, (ab)^4$.
 - (b) Prove that $\tilde{X}$ is indeed the correct one. Let $H = p_*(\tilde{X}, \tilde{x}_0)$. Prove that $H = K$.
3. HE 1.3.17
4. (HE 1.3.18) For a path-connected, locally path-connected, and semilocally simply-connected space X , call a path-connected covering space $\tilde{X} \rightarrow X$ *abelian* if it is normal and has abelian deck transformation group.
 - (a) Show that X has a *universal abelian covering space* $\tilde{X}_{ab}$, i.e. an abelian covering space that is a covering space of every other abelian covering space of X and is unique up to isomorphism.
 - (b) Describe this covering space explicitly for $X_1 = S^1 \vee S^1$ and $X_2 = S^1 \vee S^1 \vee S^1$.
5. HE 1.3.20