MAT 215A Fall 2025
Instructor: Melissa Zhang
Exam 1

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): ____________________ Name (print): ____________________

Question	Points	Score
Q1	25	
Q2	25	
Q3	25	
Q4	25	
Total:	100	

- This is a **closed-book** exam. You may not use the textbook, cheat sheets, notes, or any other outside material. No calculators, computers, phones, or any other electronics are allowed.
- You have **50 minutes** to complete this exam. If you are done early, you may leave after handing in your exam materials.
- Everyone must work on their own exam. Any suspicions of collaboration, copying, or otherwise violating the Student Code of Conduct will be forwarded to the Student Judicial Board.

Instructions

1. Use scratch paper to work out the problems.
2. Write down your solutions on the provided paper, clearly marking where your solutions for each question begins. Please write legibly and coherently; otherwise style points may be deducted.
3. Submit this exam cover sheet and your solutions to me, **in order**, so that I can paper clip them. Place your scratch paper in the pile next to me.

Comments from the instructor are written in italics.

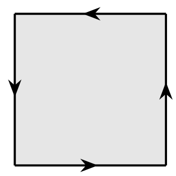
1. Let X be a topological space and let $x, y \in X$. Define $e_x : * \to X$ be the function that sends the single point in $*$ to $x \in X$; similarly define $e_y : * \to X$ to be the map that sends the point to y.

   Prove that x and y are connected by a path if and only if $e_x \simeq e_y$.

2. (a) Write down the definition of "deformation retraction."

   (b) Construct an explicit deformation retraction from $\mathbb{R}^n - \{0\}$ onto S^{n-1}.

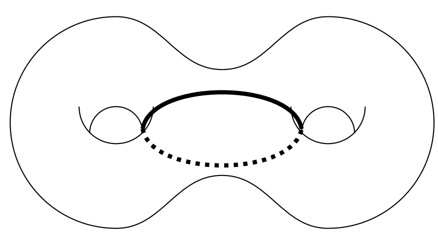
3. Let X be the space obtained by identifying the four sides of a square $[0,1]^2$ as shown:

   Observe that X is ***not*** *homeomorphic to the torus or Klein bottle; to see this, consider neighborhoods of a point in the image of the boundary of the square.*

   Write down a CW structure for X, describing the attaching maps explicitly.

4. Let X be the genus-2 surface shown below, and let A be the thick circular subspace shown. Prove that the quotient space X/A is homotopy equivalent to $S^1 \vee T^2$.

   Recall that T^2 is the torus. You may mainly draw pictures in this proof. However, you must justify why each step is a homotopy equivalence.