MAT 215B HW04

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Due Friday, 5/16/25 at 9:00 pm on Gradescope

Note The Universal Coefficient Theorem for homology is Theorem 3A.3 on page 265. In Proposition 3A.5 following Theorem 3A.3, Hatcher gives basic tools for computing Tor.

Exercise 1

(Hatcher §2.2 Ex 28) Use the Mayer-Vietoris sequence to compute the homology groups of the space obtained from a torus $S^1 \times S^1$ by attaching a Möbius band via a homeomorphism from the boundary circle of the Möbius band to the circle $S^1 \times \{x_0\}$ in the torus.

Exercise 2

Compute the reduced singular homology of $S^1 \times S^2$ using

(a) Künneth's formula

(b) the reduced Mayer-Vietoris sequence

Hint: Write $S^1 \times S^2$ as the union of two copies of $S^1 \times D^2$.

Exercise 3

(Hatcher §2.2 Ex 22) For X a finite CW complex and $p : \tilde{X} \to X$ an n-sheeted covering space, show that $\chi(\tilde{X}) = n\chi(X)$.

Exercise 4

(Hatcher §3.A Ex 1) Use the Universal Coefficient Theorem to show that if $H_*(X;\mathbb{Z})$ is finitely generated (so the Euler characteristic $\chi(X) = \sum_n (-1)^n \operatorname{rank} H_n(X;\mathbb{Z})$ is defined), then for any coefficient field F we have $\chi(X) = \sum_n (-1)^n \dim H_n(X;F)$.

You may use the following fact:

Fact. $\dim_F \operatorname{Tor}(\mathbb{Z}/m\mathbb{Z}, F) = \dim_F \mathbb{Z}/m\mathbb{Z} \otimes_{\mathbb{Z}} F$.