MAT 215B HW05

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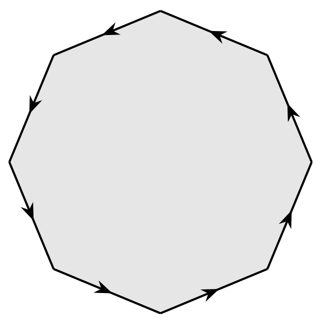
Due Friday, 5/23/25 at 9:00 pm on Gradescope

It may be helpful to give notation to duals. For example, if v is the vertex in the simplicial structure for $\mathbb{RP}^2$, then $v^* \in \mathrm{Hom}(\Delta_0(X), \mathbb{Z})$ denotes the dual: $v^*(kv) = k$, where kv means k copies of the vertex v.

Exercise 1

Consider the space X obtained by identifying all eight sides of an octagon as shown:



(a) Compute the cellular homology of X.

(b) Compute the cellular cohomology of X.

Exercise 2

(a) Compute the cellular cohomology groups of $\mathbb{RP}^2$ with $\mathbb{Z}$ and $\mathbb{Z}/2\mathbb{Z}$ coefficients.

(b) Do the same for the Klein bottle.

Exercise 3

$\mathbb{RP}^n$, or n-dimensional real projective space, is obtained by quotienting S^n by the antipodal map.

Recall that the antipodal map has degree ± 1, depending on the dimension of the sphere.

(a) Recall that there is a cell decomposition for S^n ($n \geq 0$) consisting of two cells in each dimension.

By taking quotient of this cell decomposition, describe a cell decomposition for $\mathbb{RP}^n$, and write down the cellular chain complex over $\mathbb{Z}$ coefficients.

Remember to describe the degree of the attaching maps! In effect, S^n is built inductively by adding cells onto lower dimensional spheres S^i. Your antipodal map on S^i should have degree $(-1)^{i+1}$.

(b) Compute the homology of $\mathbb{RP}^n$ over $\mathbb{Z}$ and over $\mathbb{F}_2$.