MAT 215B
Take-home final exam

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Due Wednesday, 6/11/25 at 9:00 pm on Gradescope

Instructions This is an open-notes, open-book take-home final exam. Collaboration and looking up answers on the internet are not allowed. Submit your typeset solutions to Gradescope by the due date and time. You may hand-draw any figures.

Problem 1

Use the Mayer-Vietoris sequence to show that for a space X, the suspension ΣX satisfies $\widetilde{H}_n(\Sigma X) \cong \widetilde{H}_{n-1}(X)$ for all n.

Problem 2

In this problem, you will compute the homology groups of $X = S^1 \times (S^1 \vee S^1)$ in different ways.

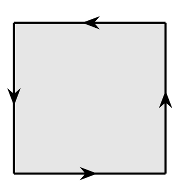
(a) Describe a cell decomposition for X. Then, write down the associated cellular chain complex, explaining clearly the associated degrees of gluing maps. Finally, compute the cellular homology of X.

(b) Compute $H_*(X)$ using the Künneth formula.

(c) Compute $H_*(X)$ using a Mayer-Vietoris sequence, by viewing X as $A \cup B$ where A and B are homotopic to tori and $A \cap B \simeq S^1$. Begin by clearly explaining what your A, B, and $A \cap B$ are.

Problem 3

Let X be the space obtained by identifying all four sides of a square, as shown in the diagram below. Compute $H_*(X;\mathbb{Z})$, $H_*(X;\mathbb{Q})$, and $H_*(X;\mathbb{Z}/2\mathbb{Z})$. Recall that this space also appeared on the midterm. Points will not be given for computing the homology of a different space.



Problem 4

Let T be a torus with meridian m, and let F be a torus with a disk deleted. Let X be the space obtained by identifying ∂F with m. Compute the homology groups of X.