MAT 215B Discussion 5

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Midterm exam on Friday, 5/2/25

The midterm will be a 50-minute, traditional pencil-and-paper exam. You will be provided with an exam packet with plenty of solution space, along with as much scratch paper (blank printer paper) as you need. You should work out your solution on scratch paper before writing down your final solution on the exam packet.

I will not be physically present during the exam. The exam problems are designed to be very clear, but if anything is unclear, write down any additional assumptions you made on the exam packet.

Corollary 2.25 For a wedge sum $\bigvee_\alpha X_\alpha$, the inclusions $\iota_\alpha : X_\alpha \hookrightarrow \bigvee_\alpha X_\alpha$ induce an isomorphism $\bigoplus_\alpha i_{\alpha*} : \bigoplus_\alpha \widetilde{H}_n(X_\alpha) \to \widetilde{H}(\bigvee_\alpha X_\alpha)$, provided that the wedge sum is formed at basepoints $x_\alpha \in X_\alpha$ where the pairs (X_α, x_α) are good.

Disclaimer: This is not meant to look like a midterm in composition. These problems do not cover all concepts that are fair game for the midterm. Also, I have not checked how long it takes to write down full solutions to these problems.

1. Using the isomorphism between $H_*^\Delta(X)$ and $H_*(X)$, compute $H_*(X)$ for the triangular parachute obtained by identifying the three vertices of Δ^2.
2. Compute the relative homology groups for the following pairs: See board for pictures.
   (a) $X = T^2$, $A =$ circle that bounds a disk in T^2
   (b) $X = T^2$, $A =$ two parallel meridional circles
   (c) $X = T^2$, $A =$ meridian union longitude
3. Compute the relative homology groups $\widetilde{H}_k(D^N, \partial D^N)$ (where $N \geq 0$).
4. Find two non-isomorphic modules A_1 and A_2 that could serve as A below make the following sequence exact:
   $$0 \to \mathbb{Z} \xrightarrow{f} A \xrightarrow{g} \mathbb{Z}/n\mathbb{Z} \to 0$$
   (For each A_i, also give the maps f, g that make the sequence exact.)
5. Prove the 'invariance of dimension' theorem:

   Theorem. If nonempty open sets $U \subset \mathbb{R}^m$ and $V \subset \mathbb{R}^n$ are homeomorphic, then $m = n$.

   Hint: First relate $(U, U - \{x\})$ and $(\mathbb{R}^m, \mathbb{R}^m - \{x\})$. Then note that $\mathbb{R}^m - \{x\} \simeq S^{m-1}$.