MAT 108 PS02

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Due Monday, April 13, 2026 at 9:00 pm on Gradescope

Instructions

There are three goals for this problem set:

- Maintain good mathematical writing form and best practices for proof-writing.
- Gain experience typesetting mathematics using LaTeX.
- Practice mathematical deductions using new techniques, including but not limited to proof by contradiction and induction.

Here are some reminders about best practices:

- For this problem set, the propositions you are instructed to prove come from the your textbook. You are only allowed to use the axioms / statements appearing before the stated proposition in the proof.
- *How much detail is needed?* In PS02, you no longer need to cite the axioms / propositions from Chapter 1. For example, it's clear to the audience (e.g. your classmates) that $-0 = 0$. On the other hand, in Chapter 2, we defined the natural numbers $\mathbb{N}$ using a set of axioms that are not very obvious to your peers. You should cite these axioms as you use them.
- You absolutely must write in full, connected English sentences. Whenever possible, do not start a sentence with mathematical symbols. Do not use symbols like $\implies$, $\forall$, $\exists$, $\therefore$, etc. (except in very specific circumstances which we will discuss in class).

Work out your proofs elsewhere first, and then make sure to give yourself enough time to typeset your solutions. Then, submit the PDF of your solutions to Gradescope by the due date and time.

- Your solution must be *neat*. You **will** be graded on style, which includes mathematical style and professionalism.
- It is your responsibility to mark where your solutions for each problem begins on Gradescope so that the TA and reader can properly grade your solutions.

Exercise 1

Prove that $1 \in \mathbb{N}$ via a proof by contradiction. (This is Proposition 2.3.) Then, deduce that that if $n \in \mathbb{N}$, then $n + 1 \in \mathbb{N}$.

Remark. The phrase *deduce that* here is indicating that the second statement follows quite immediately from the first.

Proof. □

Exercise 2

Prove Proposition 2.11:

Proposition. Let $m \in \mathbb{N}$ and $n \in \mathbb{Z}$. If $mn \in \mathbb{N}$, then $n \in \mathbb{N}$.

Proof. □

Exercise 3

(This problem will be graded for completion and style only.)

Prove the parts of Proposition 2.7 not shown in the book:

Proposition. Let $m, n, p, q \in \mathbb{Z}$.

(i) If $m < n$ then $m + p < n + p$.

(ii) If $m < n$ and $p < q$ then $m + p < n + q$.

(iv) If $m < n$ and $p < 0$ then $np < mp$.

Proof. □

Exercise 4

Use induction to prove Proposition 2.21:

Proposition. For all $k \in \mathbb{N}$, $k \geq 1$.

Proof. □

Exercise 5

Prove that there exists no integer x such that $0 < x < 1$.

Proof. □