MAT 108 PS01

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Due Monday, April 6, 2026 at 9:00 pm on Gradescope

Instructions

The goal of this problem set is to develop a solid foundation in arguing and writing mathematics. While there are very few problems, half of your focus should be on how to write mathematics clearly, using the conventions and standards of this course.

- When you begin writing proofs, it should feel like you are putting together a puzzle. You have a list of known facts (either axioms or statements we've already proven from those axioms), and your job is to fit them together into a coherent argument proving a new statement.
- For this problem set, the propositions you are instructed to prove come from the your textbook. You are only allowed to use the axioms / statements appearing before the stated proposition in the proof.
- *How much detail is needed?* For this problem set, your solutions should be at about the same level of detail as the proof of Proposition 1.6 in the book.
- You absolutely must write in full, connected English sentences. Whenever possible, do not start a sentence with mathematical symbols. Do not use symbols like $\implies$, $\forall$, $\exists$, $\therefore$, etc.

Either handwrite your typeset[1] your solutions, and submit your solutions to Gradescope by the due date and time.

- Your solution must be *neat*. You **will** be graded on style, which includes mathematical style and professionalism.
- It is your responsibility to mark where your solutions for each problem begins on Gradescope so that the TA and reader can properly grade your solutions.

[1] We will discuss how to typeset mathematics using TeX later in this course.

Exercise 1

(a) Read "Notes for the Student" (pages xv-xvi).

(b) Read Chapter 1. Make note of the notations, sentence structure, and syntax used in mathematical arguments. Also note how previous axioms or results are referenced in later proofs.

Exercise 2

Prove Proposition 1.14:

Proposition. For all $m \in \mathbb{Z}$, $m \cdot 0 = 0 = 0 \cdot m$.

Reminder. To prove an equation holds, we start at one end and use a chain of known equalities to arrive at the other end.

Exercise 3

(a) Prove Proposition 1.24:

Proposition. Let $x \in \mathbb{Z}$. If $x \cdot x = x$, then $x = 0$ or $x = 1$.

(b) Prove Proposition 1.26:

Proposition. Let $m, n \in \mathbb{Z}$. If $m \cdot n = 0$, then $m = 0$ or $n = 0$.

Exercise 4

In this exercise, we first define divisibility using only the multiplication operation $\cdot$ on $\mathbb{Z}$:

Definition. Let $m, n \in \mathbb{Z}$. If there exists a $q \in \mathbb{Z}$ such that $m = n \cdot q$, then we say m *is divisible by* n, or equivalently, n *divides* m. We denote this relationship by $n \,|\, m$.

Prove the following statements.

(a) 0 is divisible by every integer.

(b) If m is an integer not equal to 0, then m is not divisible by 0.

(c) Let $x \in \mathbb{Z}$. If x has the property that for all $m \in \mathbb{Z}$, $mx = m$, then $x = 1$.

Hint. In higher math courses, it is often beneficial to read the book sections in addition to going to lecture. In particular, both lecture and the book are pointedly useful for part (c) of the above exercise.