MAT 108 PS06

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Due Monday, May 11, 2026 at 9:00 pm on Gradescope

Instructions

There are three goals for this problem set:

- Maintain good mathematical writing form, as always.
- Review the types of arguments we made back in Chapter 1, but now for the real numbers $\mathbb{R}$ rather than the integers $\mathbb{Z}$.
- Practice working with the concepts of maximum, minimum, supremum, and infimum by proving some fundamental properties we discussed in class.

Here are some reminders about best practices:

- You absolutely must write in full, connected English sentences. Whenever possible, do not start a sentence with mathematical symbols. Do not use symbols like $\Rightarrow$, $\forall$, $\exists$, $\therefore$, etc. (except when explicitly discussing statements about logic).
- Your solution must be *neat*. You **will** be graded on style, which includes mathematical style and professionalism.
- It is your responsibility to mark where your solutions for each problem begins on Gradescope so that the TA and reader can properly grade your solutions.

Exercise 1

Prove that for all $x \in \mathbb{R}$, $x^2 < x^3$ if and only if $x > 1$.

Exercise 2

Suppose $A \subseteq B \subseteq \mathbb{R}$.

(a) Assuming the Completeness Axiom (as we always do), prove that if B is bounded above, then $\sup(A) \leq \sup(B)$.

(b) Prove that if B is bounded below, then $\inf(A) \geq \inf(B)$. (First prove Proposition 8.53 using the Completeness Axiom.)

Exercise 3

Let $A \subseteq \mathbb{R}$. Prove that if $\max(A)$ exists, then $\max(A) = \sup(A)$.

Remark. In particular, $\sup(A)$ exists, and $\sup(A) \in A$. You can also use a very similar argument to prove the analogous statement for $\min(A)$ and $\inf(A)$.