Discussion Mock Quiz Problem 1

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This is a mock exam problem, which we'll call a *mock quiz*, to help you understand how you're being evaluated, i.e. what we want to emphasize in this class.

Exercise 1

(10 imaginary points) Prove Proposition 1.13:

Proposition. Let $x \in \mathbb{Z}$. If x has the property that there exists an integer m such that

$$m + x = m,$$

then $x = 0$.

Rules: You don't need to know (and never need to memorize) the numerical labels for the axioms / propositions (e.g. Axiom 1.1(iv)) but you do need to use terms like *commutativity, associativity* of *addition, multiplication, divisibility, cancellation, additive/multiplicative inverse*, etc. to justify your proof, at least for this quiz problem. Later on we'll elide these properties for integers, but the terms will reappear when we talk about commutative rings in general.

Proof. Suppose we have $x, m \in \mathbb{Z}$ such that $m + x = m$. Adding the additive inverse $(-m)$ of m to both sides of the equation, we get

$$(-m) + (m + x) = (-m) + m = 0.$$

By associativity of addition, the left-hand side becomes $((-m)+m)+x = x$. Hence $x = 0$. $\square$

Here is an example rubric.

Keep in mind that this is a rubric for this first mock quiz problem. The rubric will necessarily change for exercises involving more advanced proofs; you will see how this changes as we advance through the course.

- (4 points) The proof is written in full sentences with proper mathematical notation. There is a square at the end of the proof.
- (2 points) Appealed to additive inverse.
- (2 points) Appealed to associativity of addition.
- (2 points) Argument is coherent and logically moves from the assumptions to the conclusion.