

# Discussion Mock Quiz Problem

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## Exercise 1

(10 imaginary points) Let  $A, B, C$  be sets. Determine whether the following statement is true or false:

$$A \cap (B - C) = (A \cap B) - (A \cap C)$$

If true, prove the claim. If false, provide a counterexample.

**SOLUTION.** The statement is true. To prove this, we show double inclusion.

First suppose  $x \in A \cap (B - C)$ . Then  $x \in A$  and  $x \in B$  but (and)  $x \notin C$ . Since  $x \in A$  and  $x \in B$ , we have  $x \in A \cap B$ . Since  $x \notin C$ , in particular  $x \notin A \cap C$ . Now we have  $x \in A \cap B$  and  $x \notin A \cap C$ , meaning that  $x \in (A \cap B) - (A \cap C)$ .

Conversely, suppose that  $x \in (A \cap B) - (A \cap C)$ . This tells us the following:

- (a)  $x \in A \cap B$ , i.e.  $x \in A$  and  $x \in B$ , but
- (b)  $x \notin A \cap C$ ; since we already know  $x \in A$ , this tells us  $x \notin C$ .

To summarize, we now have  $x \in A$ ,  $x \in B$ , and  $x \notin C$ , where the last two statements together show  $x \in B - C$ . Therefore  $x \in A \cap (B - C)$ . □

**Rubric:**

- 1pt Style
- 1pt correct setup
- 4pts Forward direction
- 4pts Backward direction