

Discussion Mock Quiz Problem

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Remark. Since mathematicians use $\mathbb{Z}/n\mathbb{Z}$ a lot, instead of writing the elements as equivalence classes $[a] \in \mathbb{Z}/n\mathbb{Z}$, we so just write a , where we choose the integer a to be in the list $\{0, 1, \dots, n - 1\}$.

However, when it's useful to think of the equivalence classes as sets, we stick with the notation $[a]$.

Exercise 1

(10 imaginary points) Determine whether the following functions are well-defined, and justify your answer (with either a proof or counterexample).

(a) $f : \mathbb{Z}/8\mathbb{Z} \rightarrow \mathbb{Z}/4\mathbb{Z}, [a] \mapsto [a]$

(b) $g : \mathbb{Z}/4\mathbb{Z} \rightarrow \mathbb{Z}/8\mathbb{Z}, [a] \mapsto [a]$

SOLUTION.

(a) The assignment f is a well-defined function. To see this, let $a, a' \in [a]$, so that $a \cong a' \pmod{8}$. Then $a - a' = 8j$ for some $j \in \mathbb{Z}$. Hence $a - a' = 4(2j)$, so $a \cong a' \pmod{4}$ as well.

(b) The assignment g is *not* well-defined. For example, consider $0, 4 \in [0] \in \mathbb{Z}/4\mathbb{Z}$. While $0 \cong 4 \pmod{4}$, modulo 8 they are not equal, since $4 - 0 = 4$ is not divisible by 8. Therefore the equivalence classes $[0], [4] \in \mathbb{Z}/8\mathbb{Z}$ are not equal.

Rubric:

- 1pt: Style
- 4pts: (a) is well-defined
- 5pts: (b) is not well-defined