MAT 108 Spring 2026
**Exam 01**

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else. I understand that any suspicions of collaboration, cheating, or otherwise violating the Student Code of Conduct will be forwarded to the Student Judicial Board.

Name (sign): _____ Name (print): _____

| Problem | Points Possible | Points Earned |
|---|---|---|
| Q1 | 15 | |
| Q1 style | 5 | |
| Q2 | 15 | |
| Q2 style | 5 | |
| Q3 | 15 | |
| Q3 style | 5 | |
| Total: | 60 | |

- You have **45 minutes** to complete this exam. If you are done early, you may leave after handing me your exam packet.
- Tear off the last page of this packet to use as scratch paper. It is **highly recommended** that you use scratch paper to figure out how you want to structure your proof, and then write down your final answer in the test packet.
- **You will be graded on both mathematical reasoning and on writing style.** In particular, your proofs must be in full sentences and **neat**, emulating the style of a typeset mathematical document. You will only be graded on the written proofs in your exam packet.
- Per the syllabus, **no electronics** may be used at any time during the exam period. All electronics must be stored inside a bag or at the front of the classroom. **No notes, texts, or other materials are allowed.**

1. (15 points + 5 style points)

   Let $a, b \in \mathbb{N}$. Prove that

   $$2a \leq b \leq a + 3 \text{ and } b \mid 3a^2 - 1$$

   if and only if $a = 1$ and $b = 2$.

   *Hint: Start by using the inequalities to restrict the possible values of $a$.*

   *For explicit numbers $x, y$, you don't need to prove to me whether $x \mid y$. For example, you may simply assert that 5 divides 25 and that 11 does not divide 25.*

2. (15 points + 5 style points)
   Prove that for all $k \in \mathbb{N}$, $2^{2k+1} + 1$ is divisible by 3.

3. (15 points + 5 style points)

Let $R, S, T \subset X$ for some set $X$. Show that

$$R \cap (S - T) = (R \cap S) - (R \cap T).$$