MAT 108 Spring 2026
Exam 02

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else. I understand that any suspicions of collaboration, cheating, or otherwise violating the Student Code of Conduct will be forwarded to the Student Judicial Board.

Name (sign): ____________________________ Name (print): ____________________________

Problem	Points Possible	Points Earned
Q1	15	
Q1 style	5	
Q2	15	
Q2 style	5	
Q3	15	
Q3 style	5	
Total:	60	

- You have **45 minutes** to complete this exam. If you are done early, you may leave after handing me your exam packet.
- Tear off the last page of this packet to use as scratch paper. It is **highly recommended** that you use scratch paper to figure out how you want to structure your proof, and then write down your final answer in the test packet.
- **You will be graded on both mathematical reasoning and on writing style.** In particular, your proofs must be in full sentences and **neat**, emulating the style of a typeset mathematical document. You will only be graded on the written proofs in your exam packet.
- Per the syllabus, **no electronics** may be used at any time during the exam period. All electronics must be stored inside a bag or at the front of the classroom. **No notes, texts, or other materials are allowed.**

1. (15 points + 5 style points)

   Using the ε-N definition of convergence, prove that

$$\lim_{k\to\infty} \frac{k-5}{k} = 1.$$

   Scratchwork (not graded):

   Proof (graded):

2. (15 points + 5 style points)

   Define a relation on the plane $\mathbb{R} \times \mathbb{R}$ by $(x_1, y_1) \sim (x_2, y_2)$ iff $x_1 + y_1 = x_2 + y_2$. Prove that this is an **equivalence relation**, and describe the **equivalence classes**.

3. (15 points + 5 style points)

We say that two positive integers $p, q \in \mathbb{N}$ are *coprime* if for any $n \in \mathbb{Z}$,

$$\text{if } p \mid n \text{ and } q \mid n, \text{ then } pq \mid n.$$

Let p, q be coprime, and consider the assignment

$$\begin{aligned} f : \mathbb{Z}/pq\mathbb{Z} &\to \mathbb{Z}/p\mathbb{Z} \times \mathbb{Z}/q\mathbb{Z} \\ [n] &\mapsto ([n], [n]). \end{aligned}$$

Prove that f is a **well-defined** function, and that it is **injective**.