MAT 215B HW01

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Due Friday at 9 pm on Gradescope

Instructions

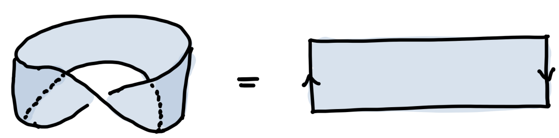
- Your solution **must be typeset**, but you may include handdrawn figures.
- Ideally, you should try all of them and then come to office hours for help on the ones you're stuck on. However, if you haven't gotten to a problem yet by the time office hours are held, I still encourage you to come to office hours.
- Some of the exercises in HWs come from Hatcher or are slightly modified versions of Hatcher exercises. For example, 'HE 2.1.1' means 'Hatcher Chapter 2 Section 1 Exercise 1'.
- Each exercise will be graded out of 3 points.

Exercise 1

(HE 2.1.1) What familiar space is the quotient Δ-complex of a 2-simplex $[v_0, v_1, v_2]$ obtained by identifying the edges $[v_0, v_1]$ and $[v_1, v_2]$, preserving the ordering of the vertices?

Exercise 2

The Möbius band M is a famous compact nonorientable surface that can be built as follows:



(a) Find a Δ-complex structure on M. You do not need to write down a verification of the defining properties of a Δ-complex.

(b) Use the Δ-complex structure from (a) to compute the simplicial homology of M.

Exercise 3

Find a Δ-complex structure for the n-dimensional sphere

$$S^n = \{p \in \mathbb{R}^{n+1} \mid |p| = 1\}$$

where $n \geq 0$. For this problem, briefly verify all the defining properties of a Δ-complex.

You may assume without proof that there is a canonical homeomorphism from the n-simplex Δ^n to the n-disk

$$D^n = \{q \in \mathbb{R}^n \mid |q| \leq 1\}.$$

Exercise 4

In class, we gave a Δ-complex structure to the torus T. Compute the simplicial homology of T explicitly, using this Δ-complex structure.

Exercise 5

Find a Δ-complex structure for the genus g closed, connected, orientable ('cco') surface Σ_g, which can be constructed by identifying edges of the $4g$-gon according to the word

$$\prod_{i=1}^{g}[a_i, b_i] = \prod_{i=1}^{g} a_i b_i a_i^{-1} b_i = a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_g b_g a_g^{-1} b_g^{-1}$$

where $g \geq 1$. (See Hatcher page 5 for a figure.)

You do not need to write down a verification of the defining properties of a Δ-complex.

Exercise 6

Find a Δ-complex structure for the nonorientable genus k closed, connected, nonorientable surface N_k, which can be constructed by identifying the edges of the $2k$-gon according to the word

$$\prod_{i=1}^{k} a_i^2 = a_1^2 a_2^2 \cdots a_k^2$$

where $\gamma \geq 1$.

You do not need to write down a verification of the defining properties of a Δ-complex.

Exercise 7

(Hatcher §2.1, exercise 9) Consider the Δ-complex X obtained from Δ^n by identifying all faces of the same dimension. Note that X has a single k-simplex for each dimension $k \leq n$.

Compute the homology groups of the Δ-complex X.