MAT 150A HW08

[add your name here]

Due Tuesday, 12/5/23 at 11:59 pm on Gradescope

Reminder. *Your homework submission* ***must be typed*** *(TeX'ed) up in full sentences, with proper mathematical formatting.*

Covered in this HW §6.7, 6.8, 6.9: group actions (i.e. group operations), orbits, stabilizers; the orbit-stabilizer theorem, counting formula

Grading Some of the (parts of) problems will be graded in detail out of several points, and necessary feedback will be given. The rest will be graded out of 2 points.

Exercise 1

Does the rule $P * A = PAP^\top$ define an operation of GL_n on $M_{n\times n}$, the set of $n \times n$ matrices? *Here, $P^\top$ is the transpose of the matrix $P \in GL_n$.*

SOLUTION.

Exercise 2

What is the stabilizer of the coset $[aH]$ for the operation fo G on G/H?

SOLUTION.

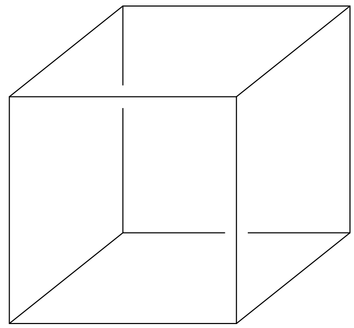
Exercise 3

Exhibit the bijective map ε from the orbit-stabilizer theorem explicitly, for the case where G is the dihedral group D_4 and S is the set of vertices of a square.

SOLUTION.

Exercise 4

A *cube* is a 3D solid with 6 square faces of equal size:



One example of the cube is the set of points $Q = [0,1]^3 \subset \mathbb{R}^3$.

Let G be the group of **rotational symmetries** of the cube. This is a subgroup of $O(3)$ consisting of *orientation-preserving* symmetries of the cube. [1]

Let V, E, and F denote the sets of vertices, edges, and faces of the cube, respectively. Check for yourself that the size of these sets are

$$|V| = 8 \quad |E| = 12 \quad |F| = 6.$$

(a) Use the counting formula to determine the order of G.

(b) Let G_v, G_e, G_f be the stabilizers of a vertex v, and edge e, and a face f of the cube. Determine the formulas of the form

$$|S| = |O_1| + |O_2| + \cdots + |O_k|$$

(formula 6.9.4 in the text) that represent the decomposition of each of the three sets V, E, F into orbits for each of the subgroups. *Your solution should contain* $9 = 3 \times 3$ *formulas, one for each (group, set) pair. First make sure you are clear on what the group and set in the group action is, in each case!*

Solution.

[1] The group of orientation-preserving isometries of $\mathbb{R}^3$ is called $SO(3)$.