LECTURE 10

Q.

$$A = \begin{pmatrix} 1 & 1.05 \\ 1 & 1 \\ 1 & 0.95 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1/\sqrt{2} \\ 1 & 0 \\ 1 & -1/\sqrt{2} \end{pmatrix}$$

Show that the colspace of A = colspace of B.

(Chapter 4 — not on exam but some are review topics / tied to previous topics, useful as review)

<u>Key concept</u> We can solve linear systems of equations
- Gauss Elim for exact soln
- Normal Equations for best fit

But in the context of signal + noise, we care about the "data quality" — in our context, how robustly a set of vectors determines the span.

eg. Nearly linearly independent columns:

$$A = \begin{pmatrix} 1 & 1.05 \\ 1 & 1 \\ 1 & 0.95 \end{pmatrix}$$

Nearly orthogonal columns:

$$B = \begin{pmatrix} 1 & 1/\sqrt{2} \\ 1 & 0 \\ 1 & -1/\sqrt{2} \end{pmatrix}$$

Same columnspace. What happens when I add some noise / small perturbation?

<u>Motto</u> "Good" basis vectors are "nearly orthogonal".

§4.1 Orthogonal Vectors & Matrices

- $x, y \neq 0 \in \mathbb{R}^n$ orthogonal if $x^T y = 0$ ie $\cos\theta(x,y) = 0$, $y^T x = 0$.

prop. Let q_j ($j \in [n]$) be orthogonal (ie mutually orthogonal). Then they're linearly independent.

Q. Converse ?

Pf. Easy – $q_n = \sum_{i<n} \alpha_i q_i \Rightarrow q_n^T q_n = 0$ ↯. ☒

- Set of orth vecs $\{q_j\}$ are normalized if they're $\|q_j\|_2 = 1 \; \forall j$.

$\Rightarrow$ they are orthonormal, and form an orthonormal basis for their span.

- a square matrix $Q = (q_1 \ldots q_m) \in \mathbb{R}^{m \times m}$ with orthonormal cols is an orthogonal matrix

Q. Why do you think I chose square matrix?

Facts (Why are these true ?)

- $Q^T Q = I$ Why?
- $Q^{-1} = Q^T$ Why?
- If Q, P orth $\Rightarrow$ PQ also ortho. Why?
- If $Q_1 \in \mathbb{R}^{m \times k}$ $(k < m)$ with orthonormal columns, then $\exists\, Q_2 \in \mathbb{R}^{m \times (m-k)}$ s.t. $Q = [Q_1 \; Q_2]$ is orthogonal matrix.

Q. How would you prove this (these)?

* What are the eigenvalues of an orthogonal matrix?