MAT 108

Exam 2

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): ___________________________ Name (print): ___________________________

Problem	Points Possible	Points Earned
Q1	15	
Q1 style	5	
Q2	15	
Q2 style	5	
Q3	15	
Q3 style	5	
Total:	60	

- You have 45 minutes to complete this exam. If you are done early, you may leave after handing me your exam packet and any scratch paper you used.
- The total number of points assigned to each problem part is shown here and within the exam.
- **You will be graded on both mathematical reasoning and on writing style.** In particular, your proofs must be in full sentences and neat, emulating the style of a typeset mathematical document.
- It is highly recommended that you use scratch paper to figure out how you want to structure your proof, and then write down your final answer in the test packet.
- You will be provided with scratch paper, and can obtain more from me during the exam. Write your name at the top of each scratch sheet. At the end of the exam, hand in both your exam packet and your scratch paper. **You will only be graded on the written proofs in your exam packet.**

Important topics and keywords for Exam 2:

- equivalence relations, $\mathbb{Z}/n\mathbb{Z}$
- functions, injective, surjective
- real numbers, upper/lower bounds
- distance functions
- convergence, limits

Mock Exam Exercises:

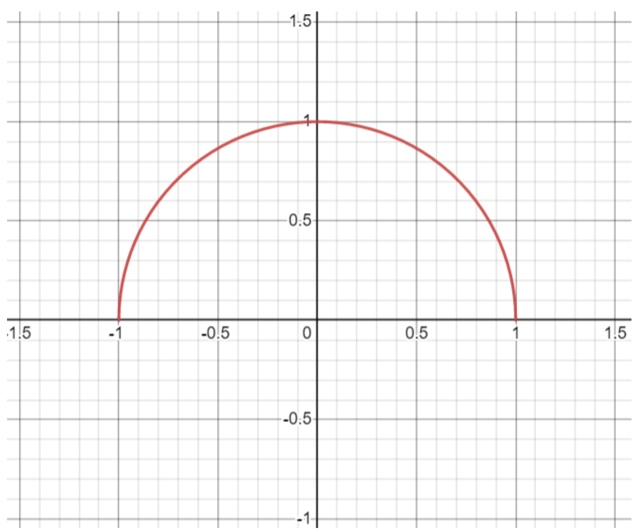
Disclaimer: The following exercises are similar in style to the ones on Exam 2, but may or may not cover the same topics. (These were discarded during the writing of the exam for one reason or another.)

1. Consider the following relation on $\mathbb{Z} \times (\mathbb{Z} - \{0\})$:

$$(m, n) \sim (s, t) \qquad \text{iff} \qquad mt = ns.$$

   Prove that $\sim$ is a well-defined *equivalence relation.*

2. Define the function $f : [-1, 1] \to \mathbb{R}$ by $x \mapsto \sqrt{1 - x^2}$, whose graph is shown below:



   Note that the slope of the graph is positive on $-1 < x < 0$ and negative on $0 < x < 1$.

   You must justify, i.e. prove, all of your answers to the following questions.

   (a) Let $I = \text{im} f$ denote the image of f. What is $\sup I$? What is $\inf I$?

   (b) Is f an injective function? Is f a surjective function?

   (c) Let (x_k) be an increasing sequence of numbers such that for all k, $0 \leq x_k \leq 1$. Prove that the sequence $(f(x_k))$ converges.