

MAT 108: Problem Set 1

(ADD NAME)

Due 1/17/23 at 11:59 pm on Canvas

Instructions. Replace the

(ADD NAME)

above with your full name.

Solve the following exercises, then typeset your solution after the

`\solution`

command at the end of each exercise. It is highly recommended that you work out the problem first before typing up your final answer. (Also, make sure you give yourself enough time to type up your solutions!)

When you are done, download the PDF of your Overleaf output, and name the file something meaningful, such as

`ps1-firstname-lastname.pdf`

and submit the file to the PS 1 Assignment on Canvas.

Reminder. *When you begin writing proofs, it should feel like you are putting together a puzzle. You a list of known facts (either axioms or statements we've already proven from those axioms), and your job is to fit them together into a coherent argument proving a new statement.*

For this homework set, the propositions you are instructed to prove come from the your textbook. You are only allowed to use the axioms / statements appearing before the stated proposition in the proof.

How much detail is needed? For this problem set, your solutions should be at the same level of detail as the proof of Proposition 1.6 in the book.

Exercise 1

Prove Proposition 1.14:

Proposition. For all $m \in \mathbb{Z}$, $m \cdot 0 = 0 = 0 \cdot m$.

Reminder. *To prove an equation holds, we start at one end and use a chain of known equalities to arrive at the other end.*

SOLUTION.

Exercise 2

a) Prove Proposition 1.24:

Proposition. Let $x \in \mathbb{Z}$. If $x \cdot x = x$, then $x = 0$ or $x = 1$.

b) Prove Proposition 1.26:

Proposition. Let $m, n \in \mathbb{Z}$. If $m \cdot n = 0$, then $m = 0$ or $n = 0$.

SOLUTION.

Exercise 3

In this exercise, we first define divisibility using only the multiplication operation \cdot on \mathbb{Z} :

Definition. Let $m, n \in \mathbb{Z}$. If there exists a $q \in \mathbb{Z}$ such that $m = n \cdot q$, then we say m is divisible by n , or equivalently, n divides m . We denote this relationship by $n \mid m$.

Prove the following statements.

a) 0 is divisible by every integer.

b) If m is an integer not equal to 0, then m is not divisible by 0.

c) Let $x \in \mathbb{Z}$. If x has the property that for all $m \in \mathbb{Z}$, $mx = m$, then $x = 1$.

Hint. In higher math courses, it is often beneficial to read the book sections in addition to going to lecture. In particular, both lecture and the book are pointedly useful for part (c) of the above exercise.

SOLUTION.