

# MAT 108: Problem Set 9

(ADD NAME)

Due 3/14/23 at 11:59 pm on Canvas

## Reminders:

- Your homework submission must be typed up in full sentences, with proper mathematical formatting. Handwritten homework submissions will receive a score of 0. Solutions containing incomplete sentences or poor formatting will lose points.
- You will receive feedback on PS8 by next Tuesday, 3/14. PS8 revisions are due Friday, 3/17 at 11:59 pm. Underneath your old solution, type

`\revisedsolution`

and then type your revised solution.

- All assignments for this course, except the final exam, must be submitted by Friday, 3/17 at 11:59 pm; no extensions will be possible beyond this time.

**Grading for this problem set** This problem set will be graded for *completion*: an honest attempt given to solve the problem will be given full marks. The solutions will be posted two days after the set is due so that you can verify your own answers.

## Exercise 1

Describe an algorithm for “counting” the countable set  $\mathbb{N}^3 = \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ . In other words, describe how one could construct a bijective function  $\mathbb{N} \rightarrow \mathbb{N}^3$ .

*Your description doesn't need to be 100% rigorous; this would take a long time to write down. However, your description needs to be clear enough so that a hypothetical classmate who hasn't thought about this problem would be able to understand how to count  $\mathbb{N}^3$ , and understand why your counting method would reach any given element in  $\mathbb{N}^3$  in finite time.*

**Hint.** Think about how we “counted”  $\mathbb{N} \times \mathbb{N}$  or  $\mathbb{Z} \times \mathbb{Z}$  in class.

**SOLUTION.**

## Exercise 2

Prove that the open interval  $(-\frac{\pi}{2}, \frac{\pi}{2})$  has the same cardinality as  $\mathbb{R}$ .

**Hint.** You need to find a bijection between the two sets. Do you know of a function from calculus class that gives a bijection  $(-\frac{\pi}{2}, \frac{\pi}{2}) \rightarrow \mathbb{R}$ ?

**Remark.** Once we know that one open interval has the same cardinality as  $\mathbb{R}$ , by scaling and translating using a linear function, we can show that any open interval  $(a, b) \subset \mathbb{R}$  (where  $a < b$ ) has the same cardinality as  $\mathbb{R}$ .

SOLUTION.

### Exercise 3

Prove that for each  $n \in \mathbb{N}$ ,  $\text{card } \mathcal{P}([n]) = \text{card } [2^n]$ .

SOLUTION.